those of the more recently introduced subjects of study. While strong faith in the validity of the claims of mathematics was apparent, yet there was evident a deep conviction that the teacher could justify himself and his subject only by intelligent, sympathetic, and earnest teaching.

G. N. ARMSTRONG,
Secretary.

BOOK REVIEWS.

SEND ALL COMMUNICATIONS TO W. H. BUSSEY, University of Minnesota.

Functions of a Complex Variable. By E. J. TOWNSEND, Professor and Head of the Department of Mathematics in the University of Illinois. Henry Holt and Company, New York, 1915. vii + 384 pages.

The present volume is one of the American Mathematical Series of which several have already appeared and of which Professor Townsend is the editor-in-chief. There are but few books in the English language that treat primarily of functions of a complex variable, and when we seek those of an elementary nature which are suitable for an introductory course, to be used by students who have had no further mathematical training than what is generally given in a first course in calculus, the number is very limited. There are the well-known texts, "*Introduction to Analytic Functions*" by Harkness and Morley and "*Theory of Functions of a Complex Variable*" by Burkhardt, translated by Rasor. A book as carefully and well written as the present volume, and which deals with such a fundamental and important field in mathematics, will undoubtedly be well received by both teachers and students.

In Chapter I there is a brief discussion of rational and irrational numbers; then follow the introduction of the complex number and the graphical representation of the same. The fundamental operations of addition, subtraction, multiplication, division, raising to powers, and the extraction of roots are explained for complex numbers, both analytically and graphically. The limitations of the graphic method for extraction of roots are illustrated by means of an example.

The beginning of Chapter II is devoted to the definitions and classifications of functions. The notions and properties of limits of sequences of real numbers are briefly stated, and the ideas are then extended to the realm of complex numbers. In article 13 several fundamental theorems relating to ordinary continuity of $f(z)$ with respect to z, uniform continuity in a region S, and uniform convergence along an arc are stated and proved.

In Chapter III differentiation and integration are taken up. Line integrals and their more general properties are discussed. The path of integration is defined as a curve having the property that it is monotone by segments of a finite number. This lends clearness to the discussions that follow, and obviates difficulties that would arise if a more general curve were chosen. Green's theorem for functions of two real variables, Cauchy's theorem, Cauchy's integral

formula, and the Cauchy-Riemann differential equations are then introduced. The chapter ends with a discussion of Laplace's differential equation and some of its applications to problems in mathematical physics.

Chapter IV is one of the longest in the book and deals with the mapping of given configurations from the Z-plane upon the W-plane and conversely. Detailed discussions are given for the simpler functions such as z^n, e^z, $\log z$, $\sin z$, $\cos z$, $\sinh z$, $\cosh z$. Functions of the above form when the argument is real are familiar to the student, but now much new information is added when the variable is complex.

In Chapter V the linear fractional transformations are taken up, beginning with the simpler forms and leading up to the general case. It is shown that the general linear fractional transformation has the properties belonging to a group. The transformations of the plane into itself due to the linear transformations are regarded as a problem in kinematics and the resulting motions are classified as parabolic, hyperbolic, elliptic, and loxodromic.

Chapter VI deals with infinite series. The more important properties of such series with complex terms are considered. The laws of operations with series are explained and illustrated with problems worked out in the text. Double series, uniform convergence, differentiation and integration of series, and power series are taken up in succession.

Chapter VII takes up the discussion of single-valued functions. It begins with a rather thorough treatment of analytic continuation. By the results thus obtained the author formulates more exactly his definition of an analytic function. Following this we have the treatment of singular points and zero points, Laurent's expansion, residues, the fundamental theorem of algebra, rational functions, transcendental functions, Mittag-Leffler's theorem, infinite products and a rather brief discussion of periodic functions.

Chapter VIII, the last in the book, treats of multiple-valued functions. Here the author points out clearly the distinction between multiple-valued analytic functions and multiple-valued expressions representing more than one single-valued analytic function. It is next shown that branch-points are characteristic of the former class. The advantage of the Riemann surface in representing multiple-valued functions is then brought out, and the surface is applied to some typical cases. After discussing some general properties of Riemann surfaces, interesting physical applications to the potential are given. Finally, in Art. 69, the general case of the algebraic function is discussed.

At the end of each chapter there is a list of well-graded exercises to be worked out by the student. It is only by the working out of a large number of such problems in connection with the theory that a student can hope to get a clear idea of the subject. Very excellent geometrical figures are given throughout the text. The typography and general appearance of the book are good. There are but few errors in printing, and those that exist should be of no serious trouble even to a beginner.

HANS H. DALAKER.

UNIVERSITY OF MINNESOTA,